

# Breakdown of the coexistence of spin-singlet superconductivity and itinerant ferromagnetism

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We discuss the possibility of coexistence of spin-singlet superconductivity and ferromagnetism in a model where the same electrons are assumed responsible for both of them. Our calculations include both zero and finite momentum pairing states with both *s*-wave and *d*-wave pairing symmetry. Under the mean-field approximation, the thermodynamic potential of the non-magnetic superconducting (SC) state is shown to be always lower than that of the superconducting ferromagnetic (SF) state. It follows that the spin-singlet SF state is energetically unfavorable, and a spin-triplet SF state is more likely to survive in metals such as UGe<sub>2</sub> and ZrZn<sub>2</sub>.

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## I. INTRODUCTION

Recently, the observation of superconductivity in ferromagnetic metals, UGe<sub>2</sub><sup>1</sup>, ZrZn<sub>2</sub><sup>2</sup> and URhGe<sup>3</sup> has renewed the interest on the coexistence of ferromagnetism and superconductivity. The investigation on the superconducting ferrromagnets traces back to the original works of Clogston<sup>4</sup>, Chandrasekhar<sup>5</sup>, and Abrikosov *et al.*<sup>6</sup>. At the same time, the possibility of a finite momentum pairing state coexisting with the long range ferromagnetic order was also revealed by Fulde and Ferrell<sup>7</sup> and by Larkin and Ovchinnikov<sup>8</sup>. This finite momentum pairing state is usually called the FFLO state. These early works<sup>4,5,6,7,8</sup> focused on the superconductivity in the metals with a spin-exchange field, such as produced by ferromagnetically aligned impurities. In such a superconducting ferromagnet, there are two kinds of electrons, respectively, responsible for ferromagnetism and superconductivity. One is the localized electrons forming a ferromagnetic background in the metal by way of an indirect exchange coupling through itinerant electrons, the other is the itinerant electrons forming the Cooper pair due to an effective attractive interaction. Therefore, the magnetic exchange energy *I* is independent of the superconducting gap  $\Delta$ . If the spin-exchange field is weak enough, the superconductivity can appear against the ferromagnetic background. The ground state of the system is determined by the ratio  $I/\Delta_0$ , where  $\Delta_0$  is the gap in a non-magnetic superconductor. In the three-dimensional *s*-wave case, for  $I/\Delta_0 \lesssim 0.707$  all of the itinerant electrons near the Fermi level form the Cooper pairs with opposite spins and the center-of-mass momentum equal to zero<sup>4</sup>, while for  $0.707 \lesssim I/\Delta_0 \lesssim 0.754$  part of the itinerant electrons near the Fermi level form the Cooper pairs with a finite center-of-mass momentum and the unpaired electrons show a finite magnetization which is the paramagnetic response of the itinerant electrons to the exchange field caused by localized spins<sup>7,8</sup>. For  $I/\Delta_0 \gtrsim 0.754$ , the formation of the Cooper pairs is totally suppressed and the superconductivity is destroyed. In the two-dimensional  $d_{x^2-y^2}$  superconductor, with the

increasing of the exchange field, the ground state of the system is changed from a zero momentum pairing state first to a finite momentum pairing state and then to a normal state, the same as in the *s*-wave case except that the zero momentum pairing  $d_{x^2-y^2}$  state in the magnetic field has a finite magnetization<sup>9</sup>.

In the recently discovered superconducting ferromagnets UGe<sub>2</sub><sup>1</sup> and ZrZn<sub>2</sub><sup>2</sup>, the superconductivity and the ferromagnetism disappear at the same critical value,  $p_c$ , under the application of the hydrostatic pressure. This feature may suggest that the same electrons are responsible for both ferromagnetism and superconductivity in these novel materials, in contrast to the conventional case of a metal with magnetic impurities. Very recently, along this direction, Karchev *et al.*<sup>10</sup> have developed a theoretical model, in which the long range ferromagnetic order does not result from an indirect exchange coupling between the localized spins, but is a consequence of a spontaneously broken spin rotation symmetry of those itinerant electrons which participate in the Cooper pair formation.

In this paper, we employ the model in Ref. 10 to discuss the possibility of the coexistence of spin-singlet superconductivity and itinerant ferromagnetism. There are two kinds of interactions between the itinerant electrons in this model. One is an attractive interaction in the Bardeen-Cooper-Schrieffer (BCS) form resulting in the superconductivity, the other is an exchange coupling resulting in the ferromagnetic order. Thus the magnetic exchange energy and the superconducting gap are related to each other and will be solved self-consistently. Under the mean-field approximation, the thermodynamic potential of the superconducting ferromagnetic (SF) state and that of the non-magnetic superconducting (SC) state are obtained analytically at zero temperature. Our calculations include both the *s*-wave and *d*-wave cases. For each pairing symmetry, both the zero momentum and the finite momentum pairing states are discussed. It is shown that the thermodynamic potential of the SF state is always higher than that of the non-magnetic SC state in all four cases. Therefore, the coexistence of ferromagnetism and spin-singlet superconductivity can not be realized if

the same electrons are assumed responsible for both of them. A spin-triplet superconductivity is more likely to survive in those novel superconducting ferromagnets such as  $\text{UGe}_2^{1,11,12}$  and  $\text{ZrZn}_2^{2,13,14,15}$ . In Sec. II to Sec. IV we will explicitly illustrate this viewpoint for each case and in Sec. V our results are briefly summarized.

## II. ZERO MOMENTUM PAIRING *s*-WAVE CASE

Our starting point is the model Hamiltonian<sup>10</sup>

$$H = \int d^3r \sum_{\sigma} c_{\sigma}^{\dagger}(\vec{r}) \left( -\frac{1}{2m^*} \vec{\nabla}^2 - \mu \right) c_{\sigma}(\vec{r}) - \frac{J}{2} \int d^3r \vec{S}(\vec{r}) \cdot \vec{S}(\vec{r}) - g \int d^3r c_{\uparrow}^{\dagger}(\vec{r}) c_{\downarrow}^{\dagger}(\vec{r}) c_{\downarrow}(\vec{r}) c_{\uparrow}(\vec{r}), \quad (1)$$

where  $c_{\sigma}(\vec{r})$  are the fermion fields with spin  $\sigma = \uparrow, \downarrow$ ,  $\vec{S} = \frac{1}{2} \sum_{\sigma, \sigma'=\uparrow, \downarrow} c_{\sigma}^{\dagger} \vec{\tau}_{\sigma \sigma'} c_{\sigma'}$  is the spin field,  $\vec{\tau}$  is the Pauli matrices,  $m^*$  is the effective mass of the electron, and  $\mu$  is the chemical potential. Hamiltonian (1) contains a kinetic energy term, a ferromagnetic exchange coupling term with strength  $J$ , and a four-fermion attractive interaction term with strength  $g$ . The coupling constant  $g$  has a finite value only in the thin shell of the width  $2\epsilon_c$  around the Fermi surface, as in the standard BCS theory. Under the mean-field approximation, Hamiltonian (1) is reduced to<sup>10</sup>

$$H_{\text{MF}} = \sum_{\vec{p}} \epsilon_p \left( c_{\vec{p}\uparrow}^{\dagger} c_{\vec{p}\uparrow} + c_{\vec{p}\downarrow}^{\dagger} c_{\vec{p}\downarrow} \right) - \frac{JM}{2} \sum_{\vec{p}} \left( c_{\vec{p}\downarrow}^{\dagger} c_{\vec{p}\downarrow} - c_{\vec{p}\uparrow}^{\dagger} c_{\vec{p}\uparrow} \right) + \frac{1}{2} JM^2 - \sum_{\vec{p}} \left( \Delta c_{\vec{p}\uparrow}^{\dagger} c_{-\vec{p}\downarrow}^{\dagger} + H.c. \right) + \frac{|\Delta|^2}{g}, \quad (2)$$

and

$$M = \frac{1}{2} \sum_{\vec{p}} \left( \langle c_{\vec{p}\downarrow}^{\dagger} c_{\vec{p}\downarrow} \rangle - \langle c_{\vec{p}\uparrow}^{\dagger} c_{\vec{p}\uparrow} \rangle \right), \quad (3)$$

$$\Delta = \sum_{\vec{p}} g \langle c_{-\vec{p}\downarrow} c_{\vec{p}\uparrow} \rangle, \quad (4)$$

where  $\epsilon_p = p^2/(2m^*) - \mu$  is the band energy measured from the chemical potential,  $\langle \dots \rangle$  represents the thermodynamic average,  $M$  defines the magnetization of the system, and  $\Delta$  is the superconducting gap. We note that the two constant terms in Eq. (2) result from the mean-field approximation,  $\Delta^2/g$  comes from the BCS interaction and  $JM^2/2$  from the exchange coupling. Here the magnetization defined in Eq. (3) arises from a spontaneously

breaking of spin rotation symmetry of the itinerant electrons, which is different from a paramagnetic response to a magnetic field caused by localized spins. Therefore, both the gap  $\Delta$  and the magnetic exchange energy  $I = JM/2$  are determined by Eqs. (3) and (4) self-consistently, unlike in the conventional case of a metal with magnetic impurities, where the exchange energy is considered as an external parameter<sup>4,5,6,7,8,9</sup>.

By means of a Bogoliubov transformation, Hamiltonian (2) can be diagonalized as

$$H_{\text{MF}} = \sum_{\vec{p}} \left( E_p^{\alpha} \alpha_{\vec{p}}^{\dagger} \alpha_{\vec{p}} + E_p^{\beta} \beta_{\vec{p}}^{\dagger} \beta_{\vec{p}} \right) + E_0 \quad (5)$$

with

$$E_0 = \sum_{\vec{p}} \left( \epsilon_p - \sqrt{\epsilon_p^2 + |\Delta|^2} \right) + \frac{|\Delta|^2}{g} + \frac{JM^2}{2}, \quad (6)$$

where  $\alpha_{\vec{p}}$  and  $\beta_{\vec{p}}$  are the Bogoliubov fermion fields with excitation energies

$$E_p^{\alpha} = \sqrt{\epsilon_p^2 + |\Delta|^2} + I, \quad (7a)$$

$$E_p^{\beta} = \sqrt{\epsilon_p^2 + |\Delta|^2} - I. \quad (7b)$$

The self-consistent equations (3) and (4) take the form

$$M = \frac{1}{2} \sum_{\vec{p}} (n_p^{\beta} - n_p^{\alpha}), \quad (8)$$

$$|\Delta| = \frac{g|\Delta|}{2} \sum_{\vec{p}} \frac{1 - n_p^{\alpha} - n_p^{\beta}}{\sqrt{\epsilon_p^2 + |\Delta|^2}}, \quad (9)$$

where  $n_p^{\alpha, \beta}$  is the momentum distribution function of the corresponding Bogoliubov fermions. At zero temperature,  $n_p^{\alpha, \beta}$  is zero for  $E_p^{\alpha, \beta} > 0$  and one for  $E_p^{\alpha, \beta} < 0$ . The thermodynamic potential at zero temperature can be obtained by averaging out the mean-field Hamiltonian (5). We take the thermodynamic potential of the ideal Fermi gas as the origin of the energy in the following discussions.

Obviously, the self-consistent Eqs. (8) and (9) have a non-magnetic SC solution with a finite gap  $\Delta_0$  and zero magnetization. By replacing the summation over the momentum  $\vec{p}$  in Eq. (9) with an integral over  $\epsilon_p$ , for the weak coupling limit  $gN(0) \ll 1$ , one can easily find that the energy gap in the non-magnetic SC solution is given by

$$\Delta_0 = 2\epsilon_c \exp \left( -\frac{1}{gN(0)} \right), \quad (10)$$

where  $N(0)$  is the density of states (DOS) at the Fermi level. Therefore, the thermodynamic potential of the non-magnetic SC state at zero temperature takes the well-known form

$$\Omega_{\text{SC}} = -\frac{1}{2} N(0) \Delta_0^2. \quad (11)$$

Next, we discuss the SF solution at zero temperature with both finite gap and finite magnetization. Here, the gap is a real number, and the self-consistent Eqs. (8) and (9) turn to

$$M = N(0) \int_0^{\sqrt{T^2 - \Delta^2}} d\epsilon, \quad (12)$$

$$\frac{1}{gN(0)} = \ln \frac{2\epsilon_c}{\Delta_0} = \int_{\sqrt{T^2 - \Delta^2}}^{\epsilon_c} \frac{d\epsilon}{\sqrt{\epsilon^2 + \Delta^2}}. \quad (13)$$

The integrals over  $\epsilon_p$  in the self-consistent equations are confined in a very thin shell around the Fermi level, and therefore we have replaced the DOS in the integrals by  $N(0)$ . It is shown that, in the SF state, only part of the itinerant electrons whose energies are in the range  $\sqrt{T^2 - \Delta^2} < |\epsilon| < \epsilon_c$  form the Cooper pairs while other electrons with energies in the range  $0 < |\epsilon| < \sqrt{T^2 - \Delta^2}$  remain unpaired. These unpaired itinerant electrons give rise to a finite spontaneous magnetization. Completing the integrals in Eqs. (12) and (13), one finds that, for  $r > 1$ , the SF solution is given by

$$I = \frac{r}{\sqrt{r^2 - 1}} \Delta, \quad \Delta = \sqrt{\frac{r-1}{r+1}} \Delta_0, \quad (14)$$

where  $r = JN(0)/2$  is the dimensionless measurement for the exchange coupling strength. Part of the electrons near the Fermi level remain unpaired so that the superconducting gap  $\Delta$  in the SF state is always less than  $\Delta_0$ . Here, the exchange energy  $I$  results from the exchange coupling between the itinerant electrons and is related to the gap  $\Delta$ , in contrast to the case of a metal with impurities. For fixed  $r$ , the ratio  $I/\Delta$  is a constant, independent of  $\Delta$ . When the gap vanishes the exchange energy also decreases to zero. Substituting the SF solutions for the magnetization and the gap into Eqs. (5) and (6) and averaging out the mean-field Hamiltonian (5), one obtains the thermodynamic potential of the SF state at zero temperature as

$$\Omega_{\text{SF}} = -\frac{1}{2} N(0) \Delta^2. \quad (15)$$

Comparing  $\Omega_{\text{SC}}$  with  $\Omega_{\text{SF}}$ , one finds that the SF state in the zero momentum pairing *s*-wave case is always energetically unfavorable due to  $\Delta < \Delta_0$ . This result can be understood by the following argument. The spontaneous magnetization in the SF state is very weak and the exchange energy  $I$  in the SF state is always less than  $\Delta_0$ . From Eq. (2) and (14) one finds that the Zeeman energy gained by the unpaired electrons can not compensate for the loss of the condensate energy in depairing the Cooper pairs. Therefore, all the itinerant electrons near the Fermi level remain paired with opposite spins, and the non-magnetic SC state is always more stable than the SF state. This feature is similar to that in the conventional case of a metal with impurities, where

for  $I/\Delta_0 \lesssim 0.707$  the itinerant electrons form a non-magnetic SC state in the presence of an exchange field of the ferromagnetic background. In the present case, the external exchange field is absent, so is the spontaneous magnetization of the itinerant electrons. As a result, the coexistence of *s*-wave superconductivity with zero momentum pairing and itinerant ferromagnetism can not be realized.

### III. FINITE MOMENTUM PAIRING *s*-WAVE CASE

In the conventional case of a metal with impurities, the formation of a condensate of finite center-of-mass momentum turns out to be more advantageous<sup>7,8</sup>. Such a finite momentum pairing state, where part of the itinerant electrons near the Fermi level form the Cooper pairs with a finite center-of-mass momentum and the unpaired electrons show a finite magnetization, is the ground state of the system when the exchange energy  $I$  is in a proper range. However, if the exchange energy is caused by the spontaneous magnetization of the itinerant electrons and thus related to the gap, the conclusion is completely different, as will be shown below.

Here, the center-of-mass momentum of the Cooper pair,  $\vec{q}$ , is not equal to zero, and thus the superconducting gap turns out to be a periodic function of the coordinates<sup>8</sup>, such as  $\Delta(\vec{r}) = \Delta \exp(i\vec{q} \cdot \vec{r})$ . The excitation energies of the Bogoliubov fermions in such finite momentum pairing state are given by<sup>16</sup>

$$E_p^\alpha = \sqrt{\epsilon_{p,q}^2 + \Delta^2} + I + \frac{1}{2} v_F q \cos \theta, \quad (16a)$$

$$E_p^\beta = \sqrt{\epsilon_{p,q}^2 + \Delta^2} - I - \frac{1}{2} v_F q \cos \theta, \quad (16b)$$

where  $\epsilon_{p,q} = (\epsilon_{\vec{p}+\vec{q}/2} + \epsilon_{-\vec{p}+\vec{q}/2})/2 = (p^2 + q^2)/(2m^*)$ ,  $v_F$  is the Fermi velocity, and  $\theta$  is the angle between the momentum  $\vec{p}$  and  $\vec{q}$ . The diagonalized Hamiltonian, the self-consistent equation of the magnetization and the gap equation have the same forms as Eqs. (5), (6), (8), and (9), respectively, provided that  $\epsilon_p$  is replaced by  $\epsilon_{p,q}$  and the excitation energies in Eqs. (7a) and (7b) are replaced by those in Eqs. (16a) and (16b). The momentum  $q$  is determined by the minimization of the thermodynamic potential, and  $v_F q/2$  is at most of the order  $\Delta_0$ <sup>7</sup>. By replacing the summation over the momentum  $\vec{p}$  with an integral over  $\epsilon = \epsilon_{p,q}/\Delta$ , the self-consistent equations at zero temperature can be written as

$$\begin{aligned} \ln \frac{\Delta}{\Delta_0} &= -\frac{1}{4} \left( \int_{E_p^\alpha < 0} \frac{dx d\epsilon}{\sqrt{\epsilon^2 + 1}} + \int_{E_p^\beta < 0} \frac{dx d\epsilon}{\sqrt{\epsilon^2 + 1}} \right) \\ &= -F_1(I', y), \end{aligned} \quad (17)$$

$$I' = \frac{1}{4} r \left( \int_{E_p^\beta < 0} dx d\epsilon - \int_{E_p^\alpha < 0} dx d\epsilon \right) = r F_2(I', y), \quad (18)$$

$$\frac{1}{N(0)\Delta^2} \frac{\partial \Omega}{\partial y} = F_3(I', y) = 0, \quad (19)$$

where  $I' = I/\Delta$ ,  $y = (v_F q/2)/\Delta$ ,  $x = \cos \theta$  and  $F_{1,2,3}$  are functions of  $I'$  and  $y$ , independent of  $\Delta$ . From Eqs. (18) and (19), one finds that  $I'$  and  $y$  are determined only by  $r$ . Hence, the ratio  $\Delta/\Delta_0$  in Eq. (17) is also a constant for fixed  $r$ . Combining with Eq. (10), one obtains

$$\frac{d(\frac{1}{g})}{d\Delta} = -N(0) \frac{1}{\Delta}. \quad (20)$$

This result is quite different from that in the conventional case where  $I$  is an external parameter and the right-hand side of Eq. (17), for fixed  $I$ , depends on the gap explicitly. The thermodynamic potential of the SF state with moving pairs is given by applying the formula<sup>16</sup>

$$\Omega_{\text{SF}} - \Omega(\Delta = 0) = \int_0^\Delta \Delta^2 \frac{d(\frac{1}{g})}{d\Delta} d\Delta, \quad (21)$$

where  $\Omega(\Delta = 0)$  is the energy when the gap vanishes. In our cases, the ratio  $I' = I/\Delta$  is a constant for fixed  $r$ . When the gap vanishes the exchange energy also decreases to zero. Therefore,  $\Omega(\Delta = 0)$  here is actually the energy of the ideal Fermi gas and is set to zero in our discussions. Completing the integral in Eq. (21), one obtains

$$\Omega_{\text{SF}} = -\frac{1}{2} N(0) \Delta^2. \quad (22)$$

The energy in the SF state with moving pairs defined in Eq. (22) takes the same form as that with immobile pairs defined in Eq. (15), but the gap functions in the two cases are different, because they are determined by two different self-consistent equations. Noting that the gap  $\Delta$  is always less than  $\Delta_0$ , we find that the thermodynamic potential of the SF state with moving pairs is still higher than that of the non-magnetic SC state. Our result is different from that in FFLO state, in which part of the itinerant electrons near the Fermi level remain unpaired and give rise to a finite magnetization in an exchange field of  $0.707 \lesssim I/\Delta_0 \lesssim 0.754$  and other electrons form the Cooper pairs with a finite center-of-mass momentum<sup>7</sup>. This is because that the Zeeman energy gained by the unpaired electrons in the SF state in the present model is less than that in FFLO state. In the conventional case, the exchange field is caused by localized spins and the exchange energy  $I$  is an external parameter. Each itinerant electron moving in the exchange field has an additional energy  $I$  or  $-I$  depending on the spin orientation of the electron. The difference in number between spin-up and spin-down electrons results in a finite magnetization  $M$ . The Zeeman energy of the system is given by  $-2IM$ . In the present model, the exchange field is due to the spontaneous magnetization of the itinerant electrons and the exchange energy  $I$  is determined self-consistently. Therefore, the additional energy  $I$  gained by the electron in the exchange field here is actually the interaction energy between this electron and other itinerant electrons. The

Zeeman energy of the system is obtained by adding the exchange energy of every electron together, but the interaction energy between the two electrons should be countered only once. Thus, the Zeeman energy here is  $-IM$ , which is only a half of that in an external field with the same  $I$ . This feature is embodied in Hamiltonian (2) via the constant term  $JM^2/2$ . Such a loss in the Zeeman energy results that the SF state with moving pairs is also energetically unfavorable and the coexistence of the itinerant ferromagnetism and the superconductivity is more difficult. As a result, if the exchange energy is caused by the spontaneous magnetization of the itinerant electrons and determined self-consistently, the  $s$ -wave non-magnetic SC state is always more stable than the  $s$ -wave SF state, no matter whether the center-of-mass momentum of the Cooper pair is zero or not.

## IV. $d$ -WAVE CASE

### A. Zero momentum pairing $d$ -wave case

We take a two-dimensional  $d_{x^2-y^2}$  superconductor for example. The gap function takes the form  $\Delta(\theta) = \Delta_d \cos(2\theta)$ , where  $\theta$  is the azimuthal angle of momentum  $\vec{p}$ . The self-consistent equation for the magnetization remains the form of Eq. (8) and the gap equation is given by<sup>9</sup>

$$\Delta_d = \frac{g\Delta_d}{2} \sum_{\vec{p}} \frac{1 - n_p^\alpha - n_p^\beta}{\sqrt{\epsilon_p^2 + \Delta_d^2 \cos^2(2\theta)}} \cos^2(2\theta), \quad (23)$$

where  $n_p^\alpha$  and  $n_p^\beta$  are the momentum distribution functions with the energies

$$E_p^\alpha = \sqrt{\epsilon_p^2 + \Delta_d^2 \cos^2(2\theta)} + I, \quad (24a)$$

$$E_p^\beta = \sqrt{\epsilon_p^2 + \Delta_d^2 \cos^2(2\theta)} - I. \quad (24b)$$

At first, we consider the non-magnetic SC solution where  $I = 0$  and thus  $n_p^\alpha = n_p^\beta = 0$ . Carrying out the summation in Eq. (23), the gap in the non-magnetic SC state is obtained by

$$\Delta_{d0} = 2.43\epsilon_c \exp\left(-\frac{2}{gN(0)}\right). \quad (25)$$

By using Eq. (21), the thermodynamic potential of the  $d_{x^2-y^2}$  non-magnetic SC state at zero temperature is given by

$$\Omega_{\text{SC}} = -\frac{1}{4} N(0) \Delta_{d0}^2. \quad (26)$$

Next, we turn to the SF solution, where only part of the itinerant electrons near the Fermi surface form the Cooper pairs while other electrons whose energies satisfy the inequality,  $E_p^\beta < 0$ , remain unpaired. Replacing

the summations over momentum  $\vec{p}$  in Eqs. (8) and (23) by the integrals over the energy  $\epsilon = \epsilon_p/\Delta_d$  and the azimuthal angle  $\theta$ , the gap equation and the self-consistent equation of the magnetization are reduced to

$$\ln \frac{\Delta_d}{\Delta_{d0}} = -\frac{1}{2\pi} \int_{E_p^\beta < 0} d\theta d\epsilon \frac{\cos^2(2\theta)}{\sqrt{\epsilon^2 + \cos^2(2\theta)}} \quad (27)$$

$$\frac{I}{\Delta_d} = \frac{1}{4\pi} r \int_{E_p^\beta < 0} d\theta d\epsilon. \quad (28)$$

From Eq. (28), it follows that the ratio  $I/\Delta_d$  is a constant for fixed  $r$ , and therefore the ratio  $\Delta_d/\Delta_{d0}$  in Eq. (27) is also a constant for fixed  $r$ . The exchange energy  $I$  is related to the gap  $\Delta_d$ , and decreases to zero as the gap vanishes. With the help of Eqs. (21) and (25), the thermodynamic potential of SF state is given by

$$\Omega_{SF} = -\frac{1}{4} N(0) \Delta_d^2. \quad (29)$$

The gap in the SF state is always less than that in the non-magnetic SC state. Hence, the  $d_{x^2-y^2}$  SF state always has higher energy than the  $d_{x^2-y^2}$  non-magnetic SC state. This result is the same as that in the  $s$ -wave case.

### B. Finite momentum pairing $d$ -wave case

In the  $d_{x^2-y^2}$  SF state with finite momentum pairing, the excitation energies of the quasi-particles take the form  $E_p^\alpha = \sqrt{\epsilon_{p,q}^2 + \Delta_d^2 \cos^2(2\theta) + I + (v_F q/2) \cos(\theta - \theta_q)}$ , and  $E_p^\beta = \sqrt{\epsilon_{p,q}^2 + \Delta_d^2 \cos^2(2\theta) - I - (v_F q/2) \cos(\theta - \theta_q)}$ , where  $\theta_q$  is the azimuthal angle of the momentum  $\vec{q}$  which is determined by the minimization of the thermodynamic potential. The self-consistent equations in the finite momentum pairing  $d$ -wave case have the same structure as that in the  $s$ -wave case. The equations for the magnetization and the momentum of the pair can be reduced to three expressions which only depend on  $I/\Delta_d$ ,  $(v_F q/2)/\Delta_d$ ,  $\theta_q$  and  $r$ . Therefore, the variables  $I/\Delta_d$ ,  $(v_F q/2)/\Delta_d$  and  $\theta_q$  are found to be constant for fixed  $r$ . The gap equation is reduced to the expression which only contains the variables  $\Delta_d/\Delta_{d0}$ ,  $I/\Delta_d$ ,  $(v_F q/2)/\Delta_d$ , and  $\theta_q$ . Thus, the ratio  $\Delta_d/\Delta_{d0}$  is also a constant for fixed  $r$ .

Combined with Eqs. (25) and (21), the thermodynamic potential of the SF state with finite momentum pairing is obtained as the same form as defined by Eq. (29) but the gap here is different from that in the zero momentum pairing case. It then follows that the energy in the  $d_{x^2-y^2}$  SF state with moving pairs is also higher than that of the non-magnetic SC state due to  $\Delta_d < \Delta_{d0}$  and therefore the SF state can not be realized.

### V. SUMMARY

In this paper, we employ a model including both an attractive interaction and an exchange coupling between the itinerant electrons to study the possibility of the co-existence of spin-singlet superconductivity and itinerant ferromagnetism. In this model the electrons responsible for the ferromagnetism and those forming the Cooper pairs are the same and thus the exchange energy between the two spin subbands is determined self-consistently, as opposed to the conventional case where the exchange energy is an external parameter. Under the mean-field approximation, the self-consistent equations of both the superconducting gap and the exchange energy are considered simultaneously, and the thermodynamic potential in the non-magnetic SC solution and that in the SF solution are obtained analytically at zero temperature. We discussed four cases including both the zero and finite momentum pairing state with both the  $s$ -wave and  $d$ -wave pairing symmetry. One finds that the exchange energy and the superconducting gap in the SF solution are related to each other. It is shown that the Zeeman energy gained by the unpaired electrons in the SF state can not compensate for the loss of the condensate energy in depairing the Cooper pairs and thus the thermodynamic potential of the SF state is always higher than that of the non-magnetic SC state in both the  $s$ -wave case and the  $d$ -wave case, no matter whether center-of-mass momentum of the pair is zero or not. Therefore, the coexistence of ferromagnetism and spin-singlet superconductivity can not be realized if the same electrons are assumed responsible for both of them. The present results indicate that a spin-triplet superconductivity is more likely to survive in those novel superconducting ferromagnets such as UGe<sub>2</sub> and ZrZn<sub>2</sub>, which are consistent with some new progress<sup>11,12,14,15</sup>.

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